Math Homework Helper

Number and Operations in Base Ten

MCC4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right.

MCC4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of the comparison.

Base Ten refers to the numbering system in common use. Take a number like 475, base ten refers to the position, the 5 is in the one's place, the 7 is in the ten's place and the 4 is in the hundred's place. **Each number is 10 times the value to the right of it**, hence the term 'base ten'.

Base 10 blocks are often used in class to help students grasp number. Base ten blocks have 'a unit' to represent one, 'a rod' to represent ten and 'a flat' to represent 100.

Example: 7 place: ten thousands' place

value: 70,000

Example: 9 place: hundreds' place

value: 900

Think:

Place is a word (name)
Value is a number (digits)

digit: number

expanded form: to show a number as the sum of the value of its digits

Example: 7,965 in expanded form is 7,000+900+60+5

Example: one thousand twelve in expanded form is 1,000 + 10 + 2

numeral: the usual way of writing a number

Example: 234,908

Example: 400+20+2 is 422

Example: Fifteen is 15.

number name: to show a number as word(s)

Example: 14,009: the number name is fourteen thousand, nine

Example: 23.09: the number name is twenty-three and nine hundredths

Example: 2.9: the number name is two and nine tenths

Comparing Whole Numbers

Symbols are used to show how the size of one number compares to another.

These symbols are < (less than), > (greater than), and = (equals).

For example, since 2 is smaller than 4 and 4 is larger than 2, we can write: 2 < 4, which says the same as 4 > 2 and of course, 4 = 4.

- To compare two whole numbers, first put them in numeral form.
- The number with more digits is greater than the other.
- If they have the same number of digits, compare the digits in the highest place value position (the leftmost digit of each number). The one having the larger digit is greater than the other.
- If those digits are the same, compare the next pair of digits (look at the numbers to the right).
- Repeat this until the pair of digits is different. The number with the larger digit is greater than the other.

Example: 402 has more digits than 42, so 402 > 42.

Example: 402 and 412 have the same number of digits.

We compare the leftmost digit of each number: 4 in each case.

Moving to the right, we compare the next two numbers: 0 and 1.

Since zero is less than one, 402 is less than 412 or 402<412.

Rounding

MCC4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

Rule One. Determine what your rounding digit is and underline it. Then look to the right side of it. If the digit is 0, 1, 2, 3, or 4 do not change the digit you underlined. All digits that are on the right hand side of the requested rounding digit will become 0.

Rule Two. Determine what your rounding digit is and underline it. Then look to the right of it. If the digit is 5, 6, 7, 8, or 9, your underlined number up by one. All digits that are on the right hand side of the requested rounding digit will become 0.

rounding to the nearest whole number (the ones' place)

<u>1</u>.25 to 1 4<u>5</u>.8 to 46 2<u>3</u>.398 to 23

rounding to the nearest ten

<u>2</u>1to 20 <u>9</u>5 to 100 4<u>4</u>1 to 440 1,1<u>2</u>5 to 1,130

rounding to the nearest <u>hundred</u>

<u>2</u>36 to 200 <u>7</u>54 to 800 <u>1,9</u>92 to 2,000 <u>1,2</u>32 to 1,200

rounding to the nearest thousand

<u>2</u>,363 to 2,000 <u>7</u>,541 to 8,000 1<u>4</u>,227 to 14,000

rounding to the nearest million

Estimation

MCC4.0A.3 Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

estimate the sum: round the numbers, then add

23	20	345	300	9,122	9,000
<u>+89</u>	<u>+90</u>	<u>+496</u>	<u>+500</u>	<u>+5,631</u>	<u>+6,000</u>
	110		800		15,000

estimate the difference: round the numbers, then subtract

45	50	745	700	9,129	9,000	
<u>-21</u>	<u>-20</u>	<u>-496</u>	<u>-500</u>	<u>-5,631</u>	<u>-6,000</u>	
	30			200		3,000

estimate the product: round the numbers and multiply

	400		1,400		90,000
<u>x 5</u>	<u>x 5</u>	<u>x18</u>	<u>x 20</u>	<u>x 349</u>	<u>x 300</u>
78	80	67	70	254	300

estimate the quotient: round the numbers and divide

Multiplication

MCC4.NBT.5 Multiply a whole number of up to four digits by a one-digit number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangle arrays, and/or area models.

Properties of Multiplication

commutative property of multiplication: the order of the factors does not change a product.

Example:
$$2 \times 4 = 4 \times 2$$

 $8 = 8$

associative property of multiplication: the way factors are grouped does not change the product.

Example:
$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

 $6 \times 4 = 2 \times 12$
 $24 = 24$

multiplicative identity property: any number multiplied by one remains the same number.

Example:
$$5 \times 1 = 5$$
 or $1 \times 5 = 5$

distributive property of multiplication: multiplying a sum by a number is the same as multiplying each addend by the number and then adding the problem.

Example:
$$2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$$

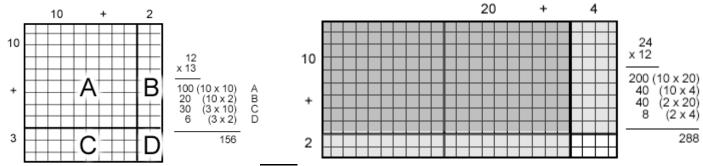
 $2 \times 7 = 6 + 8$
 $14 = 14$
Example: $432 \times 4 = (400 \times 4) + (30 \times 4) + (2 \times 4)$
 $1,728 = 1,600 + 120 + 8$
 $1,728 = 1,728$

zero property of multiplication: any number multiplied by zero will result in the product of zero

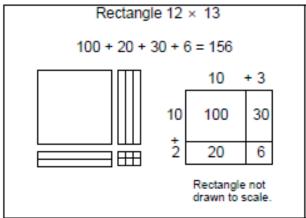
Example:
$$5 \times 0 = 0 \text{ or } 0 \times 5 = 0$$

Multiplication Algorithm

Rectangle Arrays



Area Models



Division

MCC4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Division Algorithm

Divisibility Rules (for whole numbers)

A number is <u>divisible by 2</u> if the digit in its ones' position is even, (either 0, 2, 4, 6, or 8).

A number is <u>divisible by 3</u> if the **sum** of all its digits is divisible by 3.

A number is <u>divisible by 4</u> if the number formed by the last two digits is divisible by 4.

A number is <u>divisible by 5</u> if the digit in its ones' position is 0 or 5.

A number is divisible by 6 if it is an even number and divisible by 3.

A number is <u>divisible by 8</u> if the number formed by the last three digits is divisible by 8.

A number is <u>divisible by 9</u> if the sum of all its digits is divisible by 9.

A number is <u>divisible by 10</u> if the digit in is ones' place is zero.

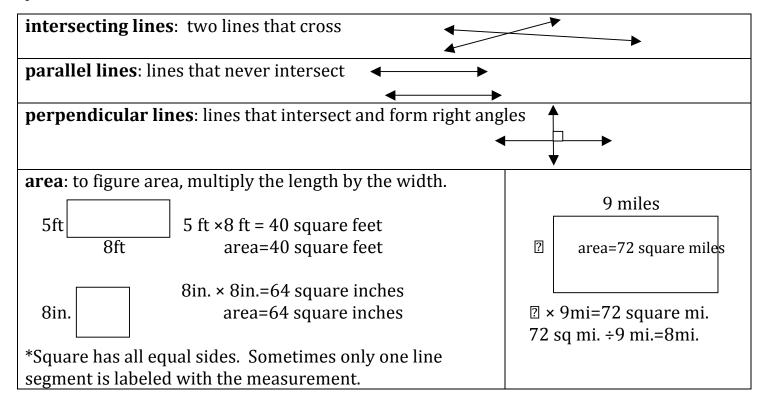
Geometry

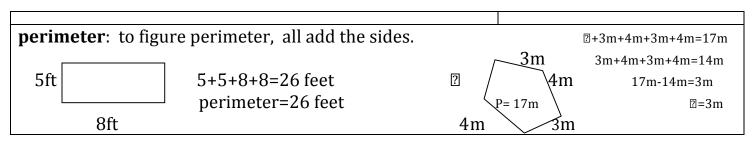
MCC4.G.1 Draw points, lines, line segments, rays, angles (acute, right, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional objects.

MCC4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

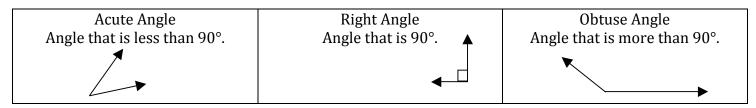
MCC4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

MCC4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

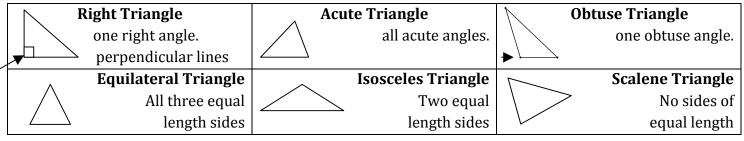


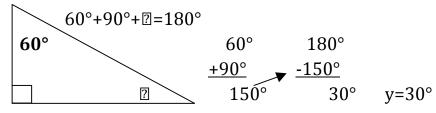


Angle: two rays that share a common endpoint.

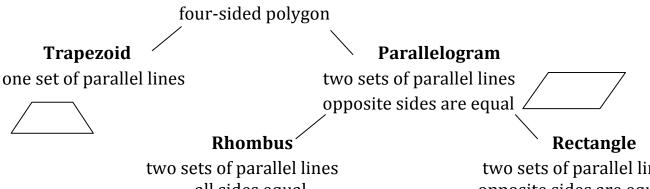


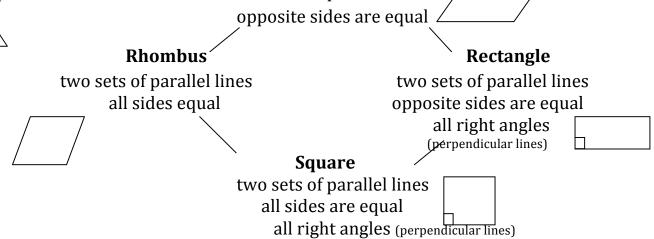
Triangle: three-sided polygon whose angles' sum are 180°.





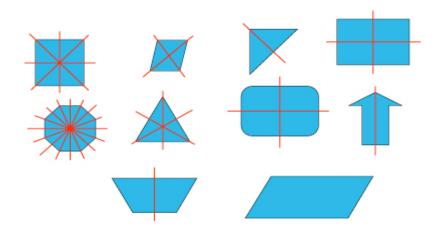
Quadrilaterals





Symmetry

- Line of symmetry is a line that divides a figure into two congruent parts, each of which is the mirror image of the other.
- When the figure having a line of symmetry is folded along the line of symmetry, the two parts should match up.



Angle Measurement

MCC4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

- **a)** An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circle arc between the points where the two rays intersect the circle.
- **b)** An angle that turns through 1/360 of a circle is called a "one-degree angle", and can be used to measure angles.

MCC4.MD.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Protractor: tool to measure angles.—

To Measure an Angle

- Find the center hole on the straight edge of the protractor.
- Place the hole over the vertex, or point, of the angle you wish to measure.
- Line up the zero on the straight edge of the protractor with one of the sides of the angle.
- Find the point where the second side of the angle crosses the curved edge of the protractor. Read the number that is written on the protractor at this point. This is the measure of the angle in degrees.

To Construct an Angle

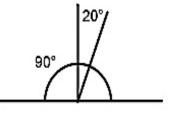
- Use the straight edge of the protractor to draw a straight line. This line will form one side of your angle.
- Find the center hole on the straight edge of the protractor. Place the hole over one end point of the line you have drawn. (continued on next page)

- Line up the zero on the straight edge of the protractor with the line.
- Make a mark at the number on the curved edge of the protractor that corresponds to the desired measure of your angle. For example, mark at 90 for a 90-degree angle.
- Use the straight edge of the protractor to connect the mark to the end point of the first line, forming an angle.

MCC4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems.

Circle Vocabulary

<u>Center:</u> the point directly in the middle of the circle.



180 - (90 + 20) = ? 180 - 110 = 70° Radius: a line segment that travels from the center of the circle to the outside rim. Radius

<u>Diameter:</u> a line segment that travels completely across the circle passing through the center. Diameter

Benchmark angles: 90° is a right angle 180° is a straight angle

Measurement

MCC4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; lb, oz; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit.

MCC4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

	metric units	customary units	
weight	1 kilogram (kg)=1,000 grams (g)	1 pound (lb)=16 ounces (oz)	
mass			
length	1 kilometer (km)=1,000 meter (m)	1 yard (yd)=3 feet (ft) or 36 in.	
distance 1 meter (m)=100 centimeters (cm)		1 foot (ft)=12 inches (in.)	
capacity	1 liter (l)=1,000 millimeters (ml)	1 gallon (gal)=4 quarts (qt)	
		1 quart (qt)=2 pints (pt)	
volume		1 pint (pt)=2 cups (c)	
time	1 hour=60 minutes		
unie	1 minute=60 seconds		

To convert a larger unit to a smaller unit of measure, you multiply. For example you change 5 meters (large unit) to centimeters (smaller unit), there are 100 centimeters in one meter. Multiply $5 \times 100 = 500$ cm. Therefore 5 m = 500cm.

Sometimes conversions are multi-step problems such as converting 2 gallons to pints. First, multiply 2 by 4, which is 8 quarts; then multiple 8 quarts by 2 which equals 16 pints.

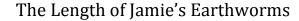
Data

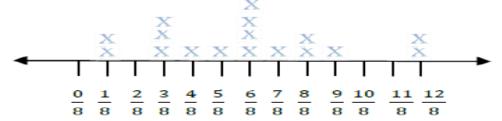
MCC.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (12, 14,18). Solve problems involving addition and subtraction of fractions by using information presented in line plots.

A **line plot** is a graph that shows frequency of data along a number line. It is best to use a line plot when comparing fewer than 25 numbers. It is a quick, simple way to organize data. A line plot consists of a horizontal number line, on which each value of a set is denoted by an x over the corresponding value on the number line. The number of x's above each score indicates how many times each value occurred in the data set.

data set:

18, 18, 38, 38, 38, 48, 58, 68, 68, 68, 68, 78, 88, 88, 98, 128, 128

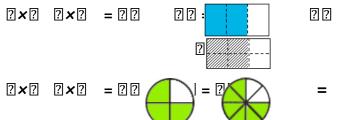




Number and Operations-Fractions

Grade 4 expectations in this domain are limited to fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 100. Extend understanding of fraction equivalence and ordering.

MCC4.NF.1 Explain why a fraction 22 is equivalent to a fraction (2×2) (2×2) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. Equivalent fractions can be created by multiplying (or dividing) both the numerator and denominator by the same number. We can do this because, if you multiply both the numerator and denominator of a fraction by the same number, the fraction remains unchanged in value. In the model below, you could get the fraction 46 by multiplying both the top and bottom of 23 by 2.



MCC4.NF.2 Compare two mactions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 12. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the

results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

<u>Build fractions from unit fractions by applying and extending previous understandings of operations on</u> whole numbers.

If two fractions have different numerators and denominators it is difficult to determine which fraction is larger. It is easier to determine which is larger if both fractions have the same denominator.

Step 1: Find a common denominator. The least common denominator of two denominators is actually the smallest number that is divisible by each of the denominators.

To find the least common denominator, simply **list the multiples** of each denominator (multiply by 2, 3, 4, etc. out to about 6 or seven usually works) then look for the **smallest number** that appears in each list.

Example: Suppose we wanted to compare *512* and *13*.

To find the least common denominator as follows:

• First we list the **multiples** of each denominator.

Multiples of **12** are **12**, 24, 36, 48, 60, ...

Multiples of **3** are 3, 6, 9, **12**, 15, 18, 21, ,...

Now, when you look at the list of multiples, you can see that **12** is the smallest number that appears in each list. Therefore, the **least common denominator** of *512* and *13* is 12.

Step 2: Multiply the numerator and denominator of one fraction by the same number so both fractions will have the same denominator.

- For example, if *512* and *13* are being compared, *13* should be multiplied by *44*. It does not change the value of *13* to be multiplied by *44* (which is equal to 1) because any number multiplied by 1 is still the same number. After the multiplication $2 \times 2 \times 2 = 2 \times 2 =$
- You may have to multiply both fractions by different numbers to produce the same denominator for both fractions. For example if 23 and 34 are compared, first we list the multiples of the two denominators.

Multiples of **3** are 3, 6, 9, **12**, 15, 18, ...

Multiples of **4** are 4, 8, **12**, 16, 20, 24,...

Next, we need to multiply 23 by 44 to get 812 and multiply 34 by 3 3 to get 912. The fraction 34 which is equal to 912 is larger than 23 which is equal to 812.

MCC4.NF.3 Understand a fraction 22 with a > 1 as a sum of fractions 1/b.

- **a)** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **b)** Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

Decomposition of Fractions

Examples: 38 = 18 + 18 + 18 or 38 = 18 + 28 218 = 1 + 1 + 18 or 88 + 88 + 18 = 2 18.

c) Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

To change 2 3/4 (a mixed number) into an equivalent fraction:

Multiply 4x2, then add 3 = 11, this is the numerator.

Use the same denominator (4), and the equivalent fraction is $^{11}/_{4}$.

Or decompose the mixed number 2 $\frac{3}{4}$ to 44 + 44 + 34 = 114 (remember 44 = 1)

d) Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction 22 as a multiple of 12.

For example, use a visual fraction model to represent 54 as the product 5×14 , recording the conclusion by the equation $54 = 5 \times 14$.

b. Understand a multiple of 22 as a multiple of 12, and use this understanding to multiply a fraction by a whole number.

For example, use a visual fraction model to express 3×25 as 6×15 recognizing this product as 65. In general, $n \times 22 = (2 \times 2)$

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

Decimals

Understand decimal notation for fractions, and compare decimal fractions.

MCC4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

For example, express 310 as 30100, and add 310 + 4100 = 34100.

110 = 0.1 or 0.10

810 = 0.8 or 0.80

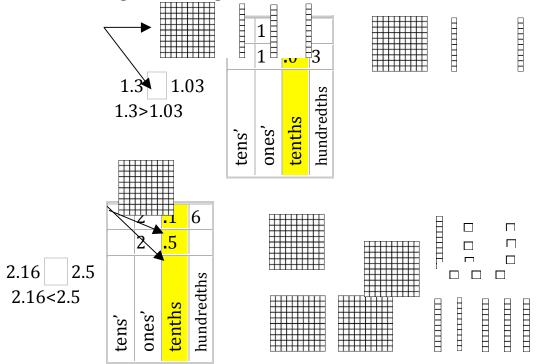
9100 = 0.0926100 = 0.26

MCC4.NF.6 Use decimal notation for fractions with denominators 10 or 100.

For example, rewrite 0.62 as 62100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

MCC4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Decimals are compared exactly the same way as whole numbers: by comparing the different place values from left to right. To help you, write the two numbers into the place value tables on top of each other. Then compare the different place values in the two numbers from left to right, starting from the biggest place value. With decimal numbers, you cannot assume the number with the most digits is the largest number. See below.



Above, the two numbers have the same value in the ones' place. Next, the decimals are compared starting with tenths place and then hundredths place (if necessary). If one decimal has a higher number in the tenths place then it is larger than a decimal with fewer tenths.

If the tenths are equal compare the hundredths until one decimal is larger or there are no more places to compare. If each decimal place value is the same then the decimals are equal.

Operations and Algebraic Thinking

Use the four operations with whole numbers to solve problems.

MCC4.0A.1 Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. See Commutative Property of Multiplication.

MCC4.0A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (See chart on the last page)

MCC4.0A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Solving for a variable (unknown quantity)in a problem

	Addition Problems	Subtraction Problems		
2 + 23 = 54	22 + 🛭 = 56	2 - 23 = 54	56 - 🛭 = 22	
54 – 23 = 31	56 – 22 = 34	54 + 23 = 77	56 – 22 = 34	
2 = 31	? = 34	? = 77	? = 34	
	Multiplication Problems		Division Problems	
② x 5 = 65	7 x 🛭 = 98	65 ÷ 🛭 = 5	98 ÷ 🛭 = 14	
$65 \div 5 = 13$	98 ÷ 7 = 14	65 ÷ 5 = 13	$98 \div 14 = 7$	
? = 13	? = 14	? = 13	? = 98	

Gain familiarity with factors and multiples.

MCC4.0A.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

<u>Factor pairs</u>: two whole numbers that multiplied together to get a product.

Multiples: a product of two given whole numbers.

multiples of **2**: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ... multiples of **8**: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...

Prime numbers: numbers with one factor pair.

Composite numbers: numbers with more than one factor pairs.

6=1×6	16=1×16	48=1×48
6=2×3	16=2×8	48=2×24
	16=4×4	48=4×12
		48=6×8

Generate and analyze patterns.

MCC4.0A.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

Standards for Mathematical Practice

Students are expected to:

1. Make sense of problems and persevere in solving them.

Students know that doing mathematics involves solving problems and discussing how they solved them.

2. Reason abstractly and quantitatively.

When problem solving, students should connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

3. Construct viable arguments and critique the reasoning of others.

Students should explain their thinking and make connections between models and equations. Students should explain their thinking to others and respond to others' thinking.

4. Model with mathematics.

Students should experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating an equation, etc.

5. Use appropriate tools strategically.

Students should consider the available tools when solving a problem and decide when certain tools might be helpful.

6. Attend to precision.

Students should use clear and precise language in explaining their reasoning with others. They careful notate units of measure and are able to state the meaning of symbols they choose.

7. Look for and make us of structure.

Students look closely to discover a pattern or structure.

8. Look for and express regularity in repeated reasoning.

Students should notice repetitive actions in computation to make generalizations. Students use models to explain calculation and understand how algorithms work.

Glossary

addend: numbers that are added together

composite number: a number with more than one factor pair.

12=1×12 2×6 3×4

<u>denominator</u> is the bottom number of the fraction; shows how many equal parts the item is divided into; a divisor

<u>difference</u>: answer to a subtraction problem

digit: number

dividend: the number that another number is divided into

Example: $45 \div 5 = 9$, 45 is the dividend

divisor: the number that another number is divided by

Example: $45 \div 5 = 9$, $\frac{5}{5}$ is the divisor

equation: state two things are the same using numbers and math symbols. An equal sign is used.

Example: 10 = 1 + 9 $8 \times 2 = 6 + 10$

equivalent fractions: have the same value, but may look different (12 = 24)

even: a number ending with 0, 2, 4, 6, or 8. Even numbers can be evenly divided into two groups.

expanded form: to show a number as the sum of the value of its digits

expression: part of a number sentence that has numbers and operation signs, but it has no equal sign

<u>factor</u>: numbers that are multiplied

factor pair: two whole numbers that multiplied together to get one product

improper fractions: have a numerator with the highest number, 54

mixed number: is a whole number and a fraction, 2 12

multiple: a product of two given whole numbers.

multiples of **8**: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, ...

number name: to show a number as word(s)
numeral: the usual way of writing a number

<u>numerator</u>: is the top number of the fraction; how many equal parts you have

odd: any number that ends with 1,3,5,7,or 9. Odd numbers cannot be evenly divided into two groups.

parallel lines: lines that never cross

perpendicular lines: lines that meet or cross at right angles (90°)

prime number: a number with one factor pair

Examples: $3 = 1 \times 3$ $17 = 1 \times 17$

product: answer to a multiplication problem

factor **x** factor = product

proper fractions: have a denominator with the highest number, 78

quotient: answer to a division problem

 $dividend \div divisor = quotient$

Example: $45 \div 5 = 9$, 9 is the quotient

remainder: the portion that will not make a complete group

Example: $21 \div 5 = 4 \text{ R1}$, 1 is the remainder

sum: answer to an addition problem

addend + addend = sum

variable: a letter or symbol that represents a number you don't know.

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Table 1. Common addition and subtraction situations. 6

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5-2=?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	Addend Unknown	Both Addends Unknown ⁷
Put Together/ Take Apart ⁸	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
			•
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare ⁹	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

⁷ These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

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Table 2. Common multiplication and division situations. 10

Group Size Unknown ("How many in each group?" Division) 3 x 6 =? There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? Arrays ¹¹ , Area ¹² Arrays ¹² , Area example. What is the area of a 3 cm by 6 cm rectangle? A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band is 6 cm long. How long will example. A rubber band is 6 cm long. How long was the rubber band at first? General A x 6 =? 3 x ? = 18, and 18 + 3 = ? 1 f 18 plums are shared equally into 3 bags, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string will you have? If 18 apples are arranged into 3 equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? A blue hat costs \$6. A red hat costs \$18 and that is 3 times as much as a blue hat costs? Measurement example. A rubber band is 6 cm long. How long was the rubber band at first. How long was the rubber band at first. Now it is stretched to be a times as long? A x b = ? A x 6 = 18, and 18 + 6 = ? 7 x 6 = 18, and 18 + 6 = ? 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will cut into 3 equal pieces of string which you will cut into 3 equal pieces. How long will example. A sequence of string which you will cut into 3 equal pieces. How long will example. A sequence of string which you will cut into 3 equal pieces. How long will act not 3 equal pieces. How long will act not 3 equal pieces. How long will her be? A red example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how lo				
There are 3 bags with 6 plums in each bag. How many plums are there in all? Equal Groups Figure Fi		Unknown Product		
each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string will you need altogether? There are 3 rows of apples with 6 apples in each row. How many apples are there? Area 127, Area 127, Area 127, Area 128 A blue hat costs 56. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rouber band is 6 cm long. How long will each piece of string will you need altogether? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? A blue hat costs 56. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it? A red hat costs 518 and that is 3 times as much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? Into 3 bags, then how many y have? Measurement example. You have 18 inches of string, which you will cut into 5 equal pieces. How long will each piece of string be? If 18 apples are arranged into 3 equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? A red hat costs 56. A red hat costs 518 and that is 3 times as much does the red hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long is the rubber band now as it was at first?		3 x 6 = ?	3 x ? = 18 , and 18 ÷ 3 = ?	? x 6 = 18 , and 18 ÷ 6 = ?
Arrays ¹¹ , Area example. What is the area of a 3 cm by 6 cm rectangle? A blue hat costs \$6. A red hat costs 3 times as much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? A page example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it? A red hat costs \$18 and that is 3 times as much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? A blue hat costs \$6. A red hat costs \$18 and that is 3 times as much does a blue hat costs. How much does a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? A red hat costs \$18 and that is 3 times as much does the red hat cost as the blue hat? Measurement example. A rubber band at first. How long was the rubber band at it was at first?		each bag. How many plums are there in all? Measurement example. You need 3 lengths of string, each 6 inches long. How much string	into 3 bags, then how many plums will be in each bag? Measurement example. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of	to a bag, then how many bags are needed? Measurement example. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces
costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long? times as much as a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first? hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long as it was at first. How long is the rubber band now as it was at first?		6 apples in each row. How many apples are there? Area example. What is the area	equal rows, how many apples will be in each row? Area example. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long	equal rows of 6 apples, how many rows will there be? Area example. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long
General a x b = ? a x ? = p , and p ÷ a = ? ? x b = p , and p ÷ b = ?	Compare	costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3	times as much as a blue hat costs. How much does a blue hat cost? Measurement example. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at	hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as
	General	a x b = ?	$a \times ? = p$, and $p \div a = ?$? x b = p, and p + b = ?

¹⁰ The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

¹¹ The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Metric

0

14

(7)

1 kilometer = 1000 meters

1 meter = 100 centimeters

1 centimeter = 10 millimeters

Customary

1 mile = 1760 yards

1 mile = 5280 feet

1 yard = 3 feet

1 foot = 12 inches

CAPACITY AND VOLUME

Metric

1 liter = 1000 milliliters

Customary

1 gallon = 4 quarts

1 gallon = 128 fluid ounces

1 quart = 2 pints

1 pint = 2 cups

1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric

1 kilogram = 1000 grams

1 gram = 1000 milligrams

Customary

1 ton = 2000 pounds

1 pound = 16 ounces

TIME

1 year = 365 days

1 year = 12 months

1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds

ω _____

Homework Helper 19

Name: